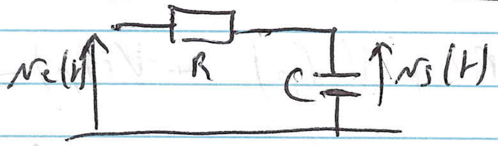
Filtre passe-bas (1$^{\text{er}}$ ordre)



$$i = C \frac{dv_s(t)}{dt}$$

$$v_s - v_e = -R.i(t) = -RC\frac{dv_s(t)}{dt}$$

$$\rightarrow \boxed{RC\frac{dv_s(t)}{dt} + v_s(t) = v_e(t)}$$

En Laplace : $RC\,p\,V_s(p) + V_s(p) = V_e(p)$

$$H(p) = \frac{V_s(p)}{V_e(p)} = \frac{1}{1+RCp}$$

$$\boxed{H(p) = \frac{1}{1+\tau p}} \quad \text{avec } \tau = RC$$

Discrétisons l'équation temporelle

On pose $\frac{dv_s(t)}{dt} \approx \frac{v_s[k] - v_s[k-1]}{T_e}$

donc $RC\frac{v_s[k]-v_s[k-1]}{T_e} + v_s[k] = v_e[k]$

donc $v_s[k]\left(1 + \frac{RC}{T_e}\right) = \frac{RC}{T_e}v_s[k-1] + v_e[k]$

$\quad$ " $\quad \left(\frac{T_e + RC}{T_e}\right)$ $\quad$ " $\quad$ "

$$v_s[k] = \frac{RC}{T_e+RC}.v_s[k-1] + \frac{T_e}{T_e+RC}v_e[k]$$

on pose $\alpha = \frac{RC}{T_e+RC}$ $\quad 1-\alpha = \frac{T_e+RC-RC}{T_e+RC} = \frac{T_e}{T_e+RC}$

alors $\boxed{v_s[k] = \alpha.v_s[k-1] + (1-\alpha).v_e[k]}$

$\alpha = \frac{\tau}{T_e+\tau}$

- si $T_e \gg \tau \Rightarrow \alpha$ proche de 0
- si $T_e \ll \tau \Rightarrow \alpha$ proche de 1

donc $0 < \alpha < 1$

Etudions dans l'espace z :

$$v_s[k] = \alpha . v_s[k-1] + (1-\alpha) . v_e[k]$$

$$Z\{v_s[k]\} = V_s(z)$$

$$Z\{v_e[k]\} = V_e(z)$$

$$Z\{v_s[k-1]\} = z^{-1} . V_s(z)$$

Rappel $Z\{y[k]\} = \sum_{k=0}^{\infty} y[k] . z^{-k}$

$$Z\{y[k-1] = \sum y[k-1] . z^{-k}$$

$$= \underbrace{y[-1] . z^0}_{k=0} + \underbrace{y[0] . z^{-1}}_{k=1}$$

$$+ y[1] . z^{-2} + \dots$$

$y[-1] = 0$ car signal causal donc

$$Z\{y[k-1]\} = 0 + y[0] . z^{-1} + y[1] . z^{-2} + \dots$$

$$= y[0] . z^0 . z^{-1} + y[1] . z^{-1} . z^{-1} + \dots$$

$$= z^{-1} . (y[0] . z^0 + y[1] . z^{-1})$$

$$\boxed{Z\{y[k-1]\} = z^{-1} . Z\{y[k]\}}$$

$$V_s(z) = \alpha\, z^{-1} V_s(z) + (1-\alpha) . V_e(z)$$

$$\boxed{H(z) = \frac{V_s(z)}{V_e(z)} = \frac{1-\alpha}{1-\alpha . z^{-1}}}$$