UVSQ ISTY Institut des Sciences et Techniques des Yvelines — université PARIS-SACLAY — CAMPUS DE MANTES EN YVELINES — CAMPUS DE SAINT-QUENTIN-EN-YVELINES	***SYSTÈMES NUMÉRIQUES***
	TD1

Boolean logic

1st exercise:

Here are some logic gate circuit problems: You can use NOT, AND and OR gates.

- Draw a logic circuit for (A + B).C.
- Draw a logic circuit for A + B.C + $\overline{D}$.
- Draw a logic circuit for A.B + $\overline{A.C}$.
- Draw a logic circuit for $\overline{(A+B)}$.(C + D).$\overline{C}$.

2nd exercise:

Here are some examples of Boolean algebra simplifications. Each line gives a form of the expression, and the rule or rules used to derive it from the previous one. Generally, there are several ways to reach the result.

- Simplify: C + $\overline{B.C}$:
- Simplify: $\overline{AB}$($\overline{A}$ + B)($\overline{B}$ + B):
- Simplify: (A + C)(AD + A$\overline{D}$) + AC + C:
- Simplify: $\overline{A}$(A + B) + (B + AA)(A + $\overline{B}$):

Note: Here is the list of rules used for the Boolean expression simplifications. This is a fairly standard list you could find most anywhere, but we thought you needed an extra copy.

The Idempotent Laws	$AA = A$		$A+A = A$	
The Associative Laws	$(AB)C = A(BC)$		$(A+B)+C = A+(B+C)$	
The Commutative Laws	$AB = BA$		$A+B = B+A$	
The Distributive Laws	$A(B+C) = AB+AC$		$A+BC = (A+B)(A+C)$	
The Identity Laws	$AF = F$	$AT = A$	$A+F = A$	$A+T = T$
The Complement Laws	$A\overline{A} = F$	$A+\overline{A} = T$	$\overline{F} = T$	$\overline{T} = F$
The Involution Law	$\overline{\overline{A}} = A$			
DeMorgan's Law	$\overline{AB} = \overline{A}+\overline{B}$		$\overline{A+B} = \overline{A}\ \overline{B}$	

Boolean Algebra Practice Problems (do not turn in):
Simplify each expression by algebraic manipulation. Try to recognize when it is appropriate to transform to the dual, simplify, and re-transform (e.g. no. 6). Try doing the problems before looking at the solutions which are at the end of this problem set.

1) $a+0=$ _____
2) $\bar{a}\cdot 0=$ _____
3) $a+\bar{a}=$ _____
4) $a+a=$ _____
5) $a+ab=$ _____
6) $a+\bar{a}b=$ _____
7) $a(\bar{a}+b)=$ _____
8) $ab+\bar{a}b=$ _____
9) $(\bar{a}+\bar{b})(\bar{a}+b)=$ _____
10) $a(a+b+c+\ldots)=$ _____

For (11),(12), (13), $f(a,b,c)=a+b+c$

11) $f(a,b,ab)=$ _____
12) $f(a,b,\bar{a}\cdot\bar{b})=$ _____
13) $f[a,b,\overline{(ab)}]=$ _____
14) $y+y\bar{y}=$ _____
15) $xy+x\bar{y}=$ _____
16) $\bar{x}+y\bar{x}=$ _____
17) $(w+\bar{x}+y+\bar{z})y=$ _____
18) $(x+\bar{y})(x+y)=$ _____
19) $w+[w+(wx)]=$ _____
20) $x[x+(xy)]=$ _____
21) $\overline{(\bar{x}+\bar{x})}=$ _____
22) $\overline{(x+\bar{x})}=$ _____
23) $w+(w\bar{x}yz)=$ _____
24) $\bar{w}\cdot\overline{(wxyz)}=$ _____
25) $xz+\bar{x}y+zy=$ _____
26) $(x+z)(\bar{x}+y)(z+y)=$ _____
27) $\bar{x}+\bar{y}+xy\bar{z}=$ _____

Problem 1: Karnaugh Maps and Minimal Expressions
For each of the following Boolean expressions, give:

i) The truth table,
ii) The Karnaugh map,
iii) The minimal sum of products expression. (Show groupings)
iv) The minimal product of sums expression. (Show groupings)

1) $(\bar{a}+b\cdot\bar{d})\cdot(c\cdot b\cdot a+\bar{c}\cdot d)$
2) $\overline{(w+\bar{x})(z\bar{y}+x)}$

***Solutions:**

1) $a + 0 = a$
2) $\bar{a} \cdot 0 = 0$
3) $a + \bar{a} = 1$
4) $a + a = a$
5) $a + ab = a(1 + b) = a$
6) $a + \bar{a}b = (a + \bar{a})(a + b) = a + b$
7) $a(\bar{a} + b) = a\bar{a} + ab = ab$
8) $ab + \bar{a}b = b(a + \bar{a}) = b$
9) $(\bar{a} + \bar{b})(\bar{a} + b) = \bar{a}\bar{a} + \bar{a}b + \bar{b}\bar{a} + \bar{b}b = \bar{a} + \bar{a}b + \bar{a}\bar{b} = \bar{a}(1 + b + \bar{b}) = \bar{a}$
10) $a(a + b + c + ...) = aa + ab + ac + ... = a + ab + ac + ... = a$
11) $f(a, b, ab) = a + b + ab = a + b$
12) $f(a, b, \bar{a} \cdot \bar{b}) = a + b + \bar{a}\bar{b} = a + b + \bar{a} = 1$
13) $f[a, b, \overline{(ab)}] = a + b + \overline{(ab)} = a + b + \bar{a} + \bar{b} = 1$
14) $y + y\bar{y} = y$
15) $xy + x\bar{y} = x(y + \bar{y}) = x$
16) $\bar{x} + y\bar{x} = \bar{x}(1 + y) = \bar{x}$
17) $(w + \bar{x} + y + \bar{z})y = y$
18) $(x + \bar{y})(x + y) = x$
19) $w + [w + (wx)] = w$
20) $x[x + (xy)] = x$
21) $\overline{(\bar{x} + \bar{x})} = x$
22) $\overline{(x + \bar{x})} = 0$
23) $w + (w\bar{x}yz) = w(1 + \bar{x}yz) = w$
24) $\bar{w} \cdot \overline{(wxyz)} = \bar{w}(\bar{w} + \bar{x} + \bar{y} + \bar{z}) = \bar{w}$
25) $xz + \bar{x}y + zy = xz + \bar{x}y$
26) $(x + z)(\bar{x} + y)(z + y) = (x + z)(\bar{x} + y) = xy + \bar{x}z$
27) $\bar{x} + \bar{y} + xy\bar{z} = \bar{x} + \bar{y} + \bar{z}$